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THEORETICAL APPLICATION AND PROOF FOR TWO-STEP INTERVAL EFFICIENCY EVALUATION METHOD

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Abstract

Regarding the "probability efficiency" in Data Envelopment Analysis (DEA), interval efficiency evaluation methods can be categorized into four types; the application of the former three types is undoubted, but no theoretical evidence has been provided to prove the fourth method. This research aims to solidify the theoretical basis for this method and utilize academic evidence to prove its rationality. After dividing the mean values of cross efficiency for decision-making units (DMU) by standard deviation, this research uses the obtained values as baseline. Besides, the research employs a theorem as theoretical proof by combining "probability formula" and cross efficiency to evaluate the weighted variables of each DMU pair. Afterwards, the highest score serves as the upper limit and the lowest as the lower limit of efficiency to illustrate the superiority of each DMU's efficiency. This can show the internal efficiency variables, so we can determine the "comprehensive efficiency" of each DMU.

Keywords: data envelopment analysis, interval efficiency evaluation, cross efficiency, CCR efficiency evaluation

(Editor's Note: Due to the complex mathematical formulas shown in this article, a single column format is used throughout in order to facilitate easier reading.)

Introduction

When the Data Envelopment Analysis (DEA) is utilized to evaluate the efficiency of decision-making units (DMU), the input and output of variables require specific data in order to obtain exact efficiency values because this technique is very sensitive. However, DMUs often encounter the issues caused by the non-transparency and inaccuracy of information and by the randomness of data occurrences. This may usually lead to low accuracy rate of the evaluated efficiency value.

Beiranvand et al. [1] evaluated sixteen American banking and financial institutions during 2003 and 2004. To deal with the problems caused by inaccurate banking data, this research adopted fuzzy Data Envelopment Analysis to pursue efficiency. In consideration of the inaccuracy factors of marketing risk management, Chen et al. [2] alternatively took SBM-DEA and fuzzy Data Envelopment Analysis in order to acquire interval efficiency value to serve as the evaluation basis for the operating efficiency of Taiwanese banks. Because the input/output data of each DMU usually comes along with randomness and inaccuracy, Tavana M. et al. [3] adopted three models (i.e. RA-FU, likelihood probability, and necessity probability) to obtain the RA-FU normal distribution of input/output data for efficiency evaluation. Tavana M. et al. [4] put forward double random constraint super-efficiency model to resolve the inaccuracy problem caused by the data source of DMU for super-efficiency model. Based on Karsak et al. [5], both quality function deployment (QFD) and fuzzy weighted average (FWA) of data envelopment analysis and innovative fuzzy multiple criteria group decision making (MCGDM) were applied to the language variables of House of Quality (HOQ) to acquire the objective weighted values of upper/lower limits to calculate its efficiency value.

Khodabakhshi M., et al. [6] employed DEA and fuzzy theory model to determine the optimism/pessimism ratio of fuzzy data and obtain the sorting methods for efficiency evaluation. Dotoli M et al. [7] investigated fifteen healthcare hospitals in Southern Italy and proposed the innovative cross efficiency fuzzy DEA efficiency evaluation method, which adopted triangular fuzzy numbers and deblurring as a trade-off approach due to the uncertainty of input/output data from DMU to assess each hospital's efficiency. Azizi et al. [8] came up with SMB loose theory to calculate the weighted intervals for the upper/lower limits of optimistism/ pessimistism. They then used the geometric mean method to calculate the efficiency interval estimates for efficiency evaluation.

All the literature above focuses on adopting fuzzy theory and innovative Data Envelopment Analysis to assess the efficiency value of DMU to handle the situation of inaccurate data sources, and thus resulted in the concept of efficiency evaluation intervals. The current research proposes "interval efficiency" probability formula for efficiency evaluation (where the efficiency value of each DMU has its own upper/lower limits, and the interval efficiency is employed to assess DMU's efficiency to address the errors of "point estimation analysis" generated by inaccurate data sources). The effects of interval efficiency of "probability formula" efficiency evaluation method are quite specific.

Experimental Methods

Although DEA enjoys outstanding performance when it is applied to assess each DMU's efficiency, it often leads to the following two error types: (1) Its efficiency evaluation values will subject to the change of DMU's input/output variables, which are difficult to obtain exact values. (2) Traditionally, this efficiency evaluation method can only determine if the efficiency of DMU A is higher than that of DMU B. However, in real circumstances, the efficiency of some parts of DMU A may be equivalent to or lower than that of DMU B.

Interval Efficiency Evaluation

To amend the errors caused by the two aforementioned traditional efficiency evaluation methods, Kang [9] put forward a "probability formula" efficiency evaluation method to further evaluate the efficiency of each DMU. In Kang's research, four instances were provided with theoretical proof: among which, the contrary relationship between merits and drawbacks were significant in three instances and posed little problem to efficiency evaluation. In the fourth instance, however, the efficiency values of DMU A were partially better or lower than those of DMU B, and it was difficult to determine which DMU had an overall predominance over the other. This research intends to offer theoretical proof for the fourth instance and evaluates the relevant merits of DMU A and DMU B in order to solidify the theoretical basis of "probability formula" efficiency evaluation method.

Theory Description

Before proving the "probability formula" theorem, it is required to determine the following definitions. According to Kang [9], the defined "efficiency interval" pairs could be divided into the following four types (see Figure 1.): where a_u is the upper limit of "efficiency interval" for DMU A, a_l is the lower limit of "efficiency interval" for DMU A, b_u is the upper limit of "efficiency interval" for DMU B, and b_l is the lower limit of "efficiency interval" for DMU B.

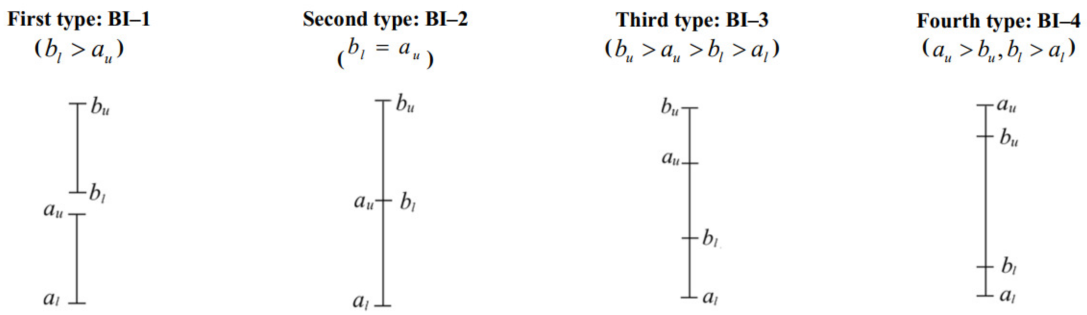

Figure 1. Four Types of "Efficiency Interval" Pairs

The relevant theorems are shown below:

Theorem:

Assume the means and the standard deviation for cross efficiency in DMU A and in DMU B are $\overline{X_A}$, S_A and $\overline{X_B}$, S_B respectively, if $\frac{\overline{X_A}}{S_A} > \frac{\overline{X_B}}{S_B}$, we can determine the interval efficiency of DMU A is better than that of DMU B.

Proof:

Assume the standard deviation and the means of Sample A and Sample B are $S_A, \overline{X_A}$, S_B and $\overline{X_B}$ respectively. To compare S_A and S_B, the coefficients of variation statistic method was adopted for comparison. Which means,

if $\frac{S_A}{\overline{X_A}} < \frac{S_B}{\overline{X_B}}$, we can determine the standard deviation of DMU A is smaller than that of DMU B. Conversely, the relationship equation should be $\frac{\overline{X_A}}{S_A} > \frac{\overline{X_B}}{S_B}$, and the value $\frac{\overline{X_A}}{S_A}$ in sample A is bigger than $\frac{\overline{X_B}}{S_B}$ in sample B. Q.E.D.

Empirical Analysis

To illustrate the interval efficiency in the research, the generation of interval efficiency needs to be explained with cross efficiency firstly.

Cross Efficiency Analysis

Take the operating efficiency evaluation of private science and technical colleges in Taiwan for example, several reference factors (including expenses, faculty number, area, staff number, graduate number, publication amount, and government subsidy) were used to assess colleges' operating efficiency, as shown in Appendix A. This research compared the efficiency of the 19 colleges with cross efficiency, and the research results are shown in Table 1.

The highest score in this table denotes the upper limit of interval efficiency for the DMU, and the lowest score denotes the lower limit of interval efficiency for the DMU in order to explicate the "interval efficiency" evaluation methods for each DMU.

To exhibit the drawbacks of CCR mode and cross efficiency in traditional DEA, the research has evaluated and ranked the operating efficiency of 19 science and technology universities in Taiwan (see Appendix A), and the results are shown in Table 2.

The interval efficiency analysis without considering the standard deviation of interval

When the standard deviation of cross efficiency is not taken into account, the ranking of interval efficiency evaluation for the 19 universities can be shown in Table 3.

Table 1. The Cross Efficiency of 19 Science and Technology Universities in Taiwan

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	0.849	1	1	0.7419	1	0.2051	0.7451	1	0.2051	0.8814	0.9403	0.9346	0.8857	1	0.8857	1	0.8841	1
2	1	1	1	0.7524	1	1	0.1349	0.7976	0.7524	0.1349	0.7681	1	0.8743	0.8614	1	0.8614	1	0.8774	0.8426
3	0.8269	0.7196	0.891	0.749	0.6486	0.8781	0.0874	0.636	0.749	0.0874	0.726	0.7865	0.8061	0.7523	0.8449	0.7523	0.8485	0.7654	0.8204
4	0.6252	0.5082	0.768	0.9705	0.4753	0.8004	0.1078	0.7594	0.9705	0.1078	0.804	0.5984	0.8464	0.8641	0.7412	0.8641	0.8441	0.8593	0.9556
5	0.7457	0.6595	0.7312	0.942	1	0.7575	0.1329	1	0.942	0.1329	0.6139	0.7038	0.6643	0.8301	0.7313	0.8301	0.9473	0.8713	0.7201
6	0.8328	0.7155	0.9558	0.7902	0.6699	1	0.1386	0.873	0.7902	0.1386	0.9262	0.781	1	1	0.9358	1	1	1	0.9803
7	0.3458	1	0.4276	0.8221	0.7573	0.6391	1	0.8718	0.8221	1	0.5563	1	0.7706	1	1	1	0.9509	0.7452	0.903
8	0.5205	0.4273	0.6142	0.8091	0.5714	0.6679	0.1119	0.9466	0.8091	0.1119	0.6636	0.4847	0.6887	0.8505	0.6053	0.8505	0.8097	0.8668	0.7469
9	0.4752	0.3519	0.5915	0.8873	0.3832	0.6345	0.0975	0.7754	0.8873	0.0975	0.7165	0.4234	0.7094	0.7997	0.5631	0.7997	0.716	0.7976	0.8281
10	0.9193	0.6346	0.8728	1	0.6686	0.9224	1	0.9554	1	1	1	0.7195	0.9285	1	0.8732	1	1	1	1
11	0.897	0.5107	1	0.8279	0.5117	1	0.0084	0.8148	0.8279	0.0084	1	0.6196	1	0.9551	0.8322	0.9551	0.9362	0.9972	1
12	0.8075	0.9035	0.9905	0.7749	0.4725	0.9577	0.1082	0.3603	0.7749	0.1082	0.7559	1	0.9091	0.7003	1	0.7003	0.8208	0.6665	0.9648
13	0.7295	0.6684	0.9242	0.6404	0.5247	0.9673	0.0966	0.7048	0.6404	0.0966	0.8808	0.7277	1	0.9311	0.8946	0.9311	0.899	0.9181	0.9402
14	0.6701	0.5118	0.7279	0.6844	0.5279	0.785	0.2084	0.827	0.6844	0.2084	0.8324	0.5686	0.8202	0.8852	0.7211	0.8852	0.8284	0.8817	0.8194
15	0.8122	0.8281	0.9194	0.7769	0.6386	1	0.1684	0.7116	0.7769	0.1684	0.882	0.89	0.9992	0.9317	1	0.9317	0.9699	0.9019	1
16	0.5981	0.4921	0.7351	0.8156	0.5577	0.8031	0.1277	1	0.8156	0.1277	0.8382	0.5578	0.869	1	0.7229	1	0.9014	1	0.9034
17	0.7528	0.6729	0.8798	0.8374	0.7989	0.9169	0.0949	0.9872	0.8374	0.0949	0.7844	0.7301	0.8772	0.9633	0.8476	0.9633	1	0.9903	0.8787
18	0.7027	0.5921	0.8571	0.8066	0.6253	0.9049	0.0985	0.9288	0.8066	0.0985	0.8504	0.662	0.9254	0.9825	0.8211	0.9825	0.9521	0.9931	0.9307
19	0.5324	0.4224	0.6897	0.8595	0.3799	0.7434	0.1131	0.7686	0.8595	0.1131	0.8443	0.5009	0.8578	0.9028	0.6692	0.9028	0.7841	0.8802	0.9636

Table 2. CCR/Cross Efficiency Evaluation and Ranking

University	CCRθ	Ranking for CCRθ	Cross Efficiency	Ranking for Cross Efficiency
DMU1	1	1	0.850428	2
DMU2	1	1	0.824075	3
DMU3	0.891	17	0.773802	8
DMU4	1	1	0.703968	15
DMU5	1	1	0.784639	7
DMU6	1	1	0.742916	10
DMU7	1	1	0.607101	19
DMU8	0.947	16	0.764238	9
DMU9	0.887	18	0.70896	14
DMU10	1	1	0.72505	13
DMU11	1	1	0.639809	18
DMU12	1	1	0.673014	17
DMU13	1	1	0.729755	12
DMU14	0.885	19	0.734508	11
DMU15	1	1	0.817257	5
DMU16	1	1	0.805618	6
DMU17	1	1	0.688282	16
DMU18	0.993	14	0.821668	4
DMU19	0.964	15	0.920752	1

Table 3. The Interval Efficiency Evaluation and Ranking Without Considering The Standard Deviation of Cross Efficiency

University	Interval Efficiency	Ranking by Probability Formula
DMU10	[0.635,1.000]	1
DMU7	[0.346,1.000]	2
DMU17	[0.208,1.000]	3
DMU1	[0.205,1.000]	4
DMU15	[0.168,1.000]	5
DMU6	[0.139,1.000]	6
DMU2	[0.135,1.000]	7
DMU5	[0.133,1.000]	8
DMU16	[0.128,1.000]	9
DMU4	[0.108,1.000]	10
DMU12	[0.108,1.000]	10
DMU13	[0.097,1.000]	12
DMU14	[0.208,0.885]	13
DMU18	[0.098,0.993]	14
DMU19	[0.113,0.964]	15
DMU8	[0.112,0.947]	16
DMU11	[0.008,1.000]	17
DMU9	[0.098,0.887]	18
DMU3	[0.087,0.891]	19

As shown in Table 3, the width of each interval efficiency is different, which means the standard deviation of each interval efficiency is not the same. Consequently, if the mean of each interval efficiency serves as the standard for efficiency evaluation, the variation of each interval will be ignored.

The interval efficiency analysis which considers the standard deviation of interval

The research not only adopted the interval efficiency of cross efficiency but especially emphasized the standard deviation of each cross efficiency. By combining the interval efficiency of cross efficiency with the standard deviation of each interval for the 19 universities, the evaluation will rank interval efficiency based on the mean of cross efficiency divided by standard deviation. The results are shown in Table 4. For better understanding, the interval efficiency of the 19 universities in Table 4 is mapped into Figure 2.

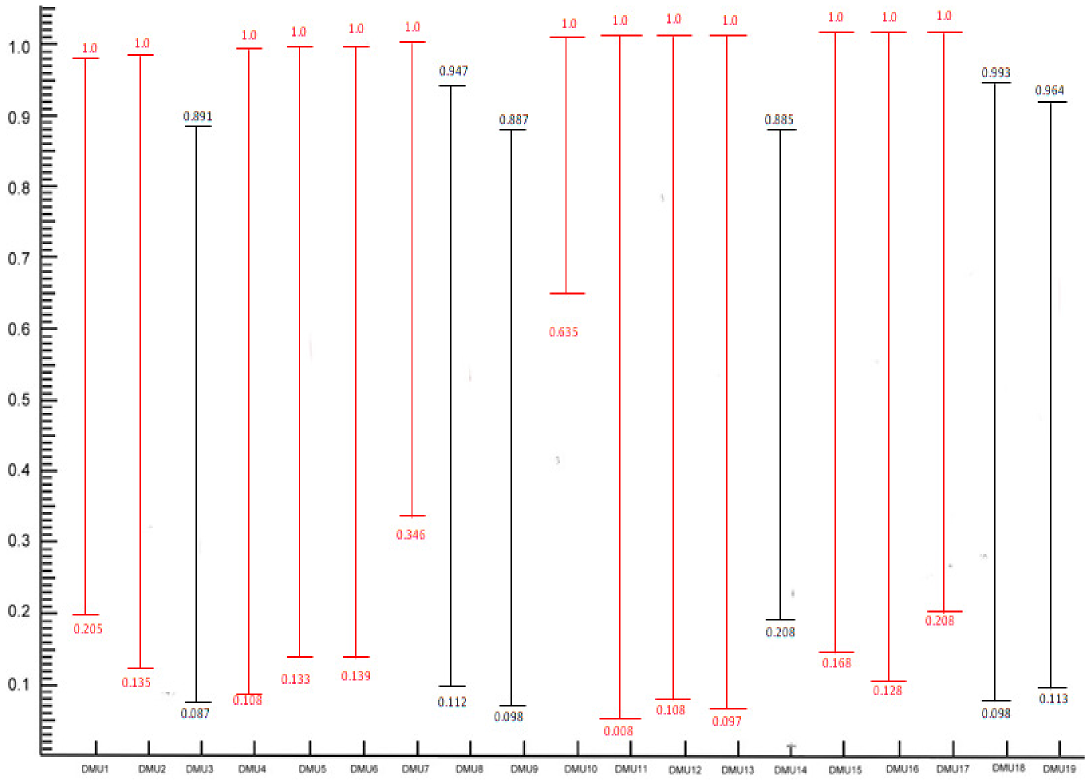

Figure 2. Interval Efficiency for 19 Universities

According to the probability evaluation without considering the standard deviation of interval efficiency, the efficiency level of each DMU in DMU7 and DMU10 is obvious, whereas the efficiency evaluation of other universities still requires comparing in pairs to obtain the ranking of efficiency evaluation. The four interval efficiency methods for probability evaluation (excluding the standard deviation of each interval efficiency) are shown below.

BI–1: This situation type does not occur in this empirical analysis.

BI–2: This situation type does not occur in this empirical analysis.

BI–3: As revealed in this empirical analysis, this situation type occurs across 13 universities, including DMU1, DMU2, DMU4, DMU5, DMU6, DMU7, DMU10, DMU11, DMU12, DMU15, DMU16, DMU17 and DMU19. As indicated in Figure 3., we can clearly determine the ranking of interval efficiency level for the 13 universities.

As shown above, the efficiency level for the interval efficiency of BI–1, BI–2 and BI–3 is rather distinguished. However, to assess the efficiency level for BI–4, the values of interval efficiency need to be converted with the mean of cross efficiency divided by standard deviation formula in order to evaluate the efficiency level of each DMU, as shown in Table 4.

BI–4: As shown in Table 4, the following university pairs fall into the fourth type "BI–4".

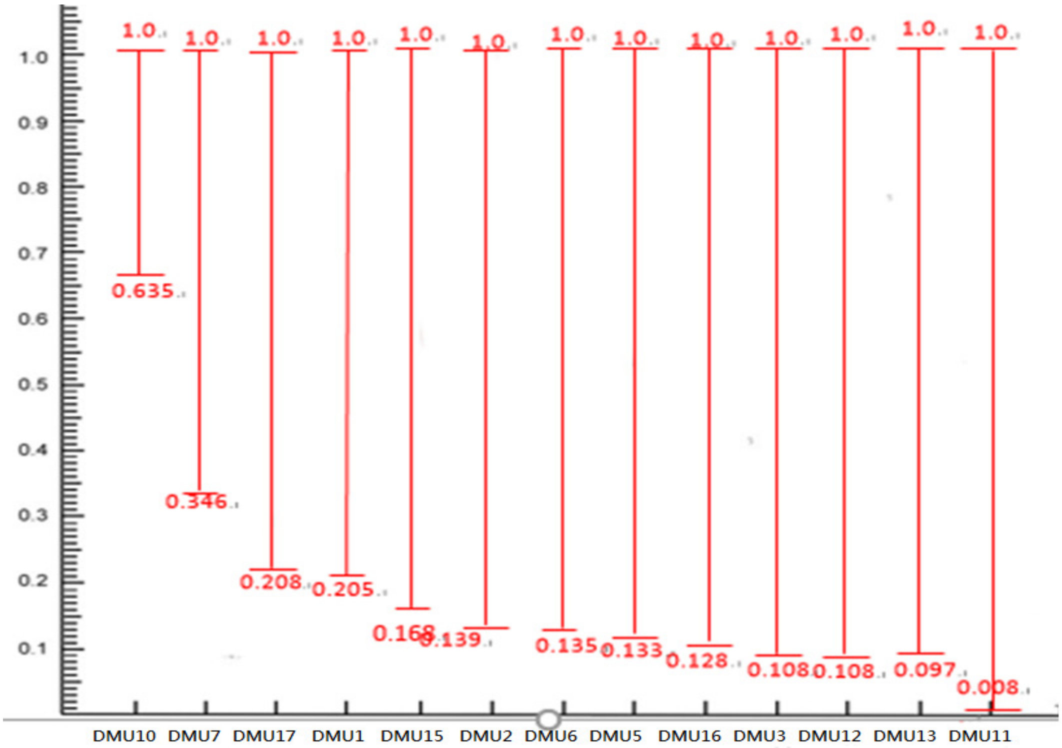

Figure 3. 13 Types of Interval Efficiency for "BI–3"

Table 4. Mean of Interval Efficiency Divided by Standard Deviation and Ranking for The 19 Universities

University	Interval Efficiency	Mean of Cross Efficiency Divided by Standard Deviation	Ranking
DMU10	[0.635,1.000]	7.742	1
DMU7	[0.346,1.000]	4.035	2
DMU1	[0.205,1.000]	3.514	3
DMU14	[0.208,0.885]	3.364	4
DMU15	[0.168,1.000]	3.265	5
DMU2	[0.135,1.000]	3.165	6
DMU6	[0.139,1.000]	3.125	7
DMU3	[0.087,0.891]	3.092	8
DMU5	[0.133,1.000]	3.026	9
DMU17	[0.208,1.000]	3.02	10

DMU18	[0.098,0.993]	2.893	11
DMU13	[0.097,1.000]	2.8	12
DMU4	[0.108,1.000]	2.784	13
DMU16	[0.128,1.000]	2.757	14
DMU8	[0.112,0.947]	2.738	15
DMU12	[0.108,1.000]	2.615	16
DMU19	[0.113,0.964]	2.587	17
DMU9	[0.098,0.887]	2.493	18
DMU11	[0.008,1.000]	2.477	19

(1) For the interval efficiency between DMU14 and DMU3, DMU14's efficiency is higher than that of DMU3 by 0.1430. For the interval efficiency between DMU14 and DMU18, DMU14's efficiency is higher than that of DMU18 by 0.0022. For the interval efficiency between DMU14 and DMU8, DMU14's efficiency is higher than that of DMU8 by 0.0407. For the interval efficiency between DMU14 and DMU19, DMU14's efficiency is higher than that of DMU19 by 0.0188. For the interval efficiency between DMU14 and DMU9, DMU14's efficiency is higher than that of DMU9 by 0.1369. In the cases above, the results are the same to the interval efficiency evaluation results which are not divided by each interval standard deviation, and the same situation applies to all the aforementioned comparisons between DMU14 and other universities. (For more details, see Appendix B)

(2) For the interval efficiency between DMU3 and DMU9, DMU3's efficiency is higher than that of the latter by 0.0009. This result is different from the interval efficiency evaluation result which is not divided by each interval standard deviation. (For more details, see Appendix C)

(3) For the interval efficiency between DMU18 and DMU8, DMU18's efficiency is higher than that of the latter by 0.0432. This result is the same to the interval efficiency evaluation result which is not divided by each interval standard deviation. (For more details, see Appendix D)

Conclusion

Kang [9] proposed to adopt interval efficiency as efficiency evaluation methods, which can be divided into four types. It is needless to further explore the application of the former three types because the efficiency levels of DMU are quite obvious. However, the application of the fourth type did not have theoretical basis back then. The current research utilized probability efficiency to evaluate the cross efficiency of each DMU. Among which, the highest score was taken as the upper limit for the efficiency evaluation of the DMU, and the lowest score was considered as the lower limit. After the mean value of this cross efficiency being divided by the standard deviation of the DMU, the results can explain the internal variation within this interval efficiency of each DMU and thus serve as theoretical basis for this evaluation method, as shown in Table 4.

The efficiency evaluation in this research method shows the following features

The mean of cross efficiency was divided by standard deviation for each DMU. Afterwards, DMUs' operation efficiency was ranked based on the internal variables. This research method not only mitigated the drawbacks of cross efficiency, but also compensated for the inefficient discrimination caused by CCR efficiency evaluation, as shown in Table 4.

In addition to improving the drawbacks of traditional interval efficiency, this research method also remained its focus on the characteristics of interval efficiency evaluation. This interval probability efficiency evaluation method not only made comprehensive evaluation for the two DMUs, but also determined if their interval efficiency was better or worse than each other, as shown in Table 4.

Traditionally, while ranking cross efficiency without considering the standard deviation of interval efficiency, the comparison results between the operation efficiency of DMU4/ DMU12 and other 19 technology universities revealed that the two universities both ranked 10th (see Table 3). In contrast, this research divided the mean of cross efficiency by the standard deviation of the DMU's efficiency. Therefore, these two technology universities (i.e. DMU4 and DMU12) would rank 13th and 16th when the internal variables of DMU were included into the efficiency evaluation. Apparently, this evaluation method adopting the mean of cross efficiency alone to determine the superiority of operation efficiency holds stronger theoretical basis than the traditional cross efficiency evaluation.

In terms of interval efficiency, this research emphasized to evaluate each interval's efficiency based on dividing the standard deviation of each interval by its mean. According to research findings, there existed differences in cases where comparing the efficiency of intervals in BI-4 between DMU3 and DMU9 and in cases where standard deviation was not included into evaluation. After further exploration, it is uncovered the distribution of the 19 cross efficiency values is rather separate. In this regard, this research not only compensates for the insufficiency of traditional interval efficiency but also exhibit the performance of this research.

Typical DEA approaches adopt the statistic point estimation and sampled data, so its efficiency evaluation is not as objective as interval efficiency evaluation. The innovation of this research is to replace point estimation with interval efficiency.

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APPENDIX A The Ranking of The Operating Efficiency for 19 Science and Technology Universities in Taiwan

Table 5. The Operating Efficiency for 19 Science and Technology Universities in Taiwan

	Expenses (Unit: thousand)	Faculty number	Area (Unit: m²)	Staff number	Graduate number	Publication amount	Government subsidy (Unit: million)	CCRθ
DMU1	2099502	407	157428	222	3870	769	77	1
DMU2	2217395	576	244233	183	4417	749	75	1
DMU3	1685299	376	196668	166	2663	557	40	0.9121
DMU4	1415249	282	171805	229	2661	521	43	0.9913
DMU5	1106059	244	91200	106	2477	296	29	1
DMU6	1325658	375	154934	179	2900	559	50	1
DMU7	1468123	328	130400	313	3180	289	31	1
DMU8	1269598	325	133380	205	3000	378	35	0.9466
DMU9	2102960	440	218337	415	4043	675	50	0.8578
DMU10	1087733	249	92277	157	2584	429	40	1
DMU11	1038035	275	100637	143	2114	369	20	1
DMU12	1147873	197	190435	115	1020	419	47	1
DMU13	862778	269	145268	125	1530	368	32	0.9941
DMU14	917589	271	58572	147	1905	295	30	0.8517
DMU15	768218	194	102911	99	1364	315	40	1
DMU16	932847	268	111128	168	2339	375	33	1
DMU17	963844	263	132545	117	2381	353	29	1

DMU18	1174112	330	163174	174	2732	467	37	0.9934
DMU19	1044296	243	133703	218	1997	427	35	1

APPENDIX B The Evaluation of Interval Efficiency Not Divided by The Standard Deviation of Each Interval

1. The higher limit for the interval efficiency of DMU14 is lower than that of DMU3: ($\frac{0.891-0.885}{0.891-0.087}$)=0.0075;

the lower limit for the interval efficiency of DMU14 is higher than that of DMU3:($\frac{0.208-0.087}{0.891-0.087}$)=0.1505.

To compare the interval efficiency of DMU14 and DMU3, the interval efficiency of DMU14 is higher than that of DMU3 by 0.1430.

2. The higher limit for the interval efficiency of DMU14 is lower than that of DMU18: ($\frac{0.993-0.885}{0.993-0.098}$)=0.1207;

the lower limit for the interval efficiency of DMU14 is higher than that of DMU18: ($\frac{0.208-0.098}{0.993-0.098}$)=0.1229.

To compare the interval efficiency of DMU14 and DMU18, the interval efficiency of DMU14 is higher than that of DMU18 by 0.0022.

3. The higher limit for the interval efficiency of DMU14 is lower than that of DMU8: ($\frac{0.947-0.885}{0.947-0.112}$)=0.0743;

the lower limit for the interval efficiency of DMU14 is higher than that of DMU8: ($\frac{0.208-0.112}{0.947-0.112}$)=0.1150.

To compare the interval efficiency of DMU14 and DMU8, the interval efficiency of DMU14 is higher than that of DMU8 by 0.0407.

4. The higher limit for the interval efficiency of DMU14 is lower than that of DMU19: ($\frac{0.964-0.885}{0.964-0.113}$)=0.0928;

the lower limit for the interval efficiency of DMU14 is higher than that of DMU19: ($\frac{0.208-0.113}{0.964-0.113}$)=0.1116.

To compare the interval efficiency of DMU14 and DMU19, the interval efficiency of DMU14 is higher than that of DMU19 by 0.0188.

5. The higher limit for the interval efficiency of DMU14 is lower than that of DMU9: ($\frac{0.887-0.885}{0.887-0.098}$)=0.0025;

the lower limit for the interval efficiency of DMU14 is higher than that of DMU9: $(\frac{0.208-0.098}{0.887-0.098})$=0.1394.

To compare the interval efficiency of DMU14 and DMU9, the interval efficiency of DMU14 is higher than that of DMU9 by 0.1369.

APPENDIX C The Interval Efficiency Between DMU3 and DMU9

1. The upper limit for the interval efficiency of DMU3 is higher than that of DMU9: $(\frac{0.891-0.887}{0.891-0.087})$=0.005;

   the lower limit for the interval efficiency of DMU3 is lower than that of DMU9: $(\frac{0.098-0.087}{0.891-0.087})$=0.014.

To compare the interval efficiency of DMU3 and DMU9, the interval efficiency of DMU3 is lower than that of DMU9 by 0.009.

APPENDIX D The Interval Efficiency Between DMU8 and DMU18

1. The upper limit for the interval efficiency of DMU18 is higher than that of DMU8: $(\frac{0.993-0.947}{0.993-0.098})$=0.0588;

   the lower limit for the interval efficiency of DMU18 is higher than that of DMU8: $(\frac{0.112-0.098}{0.993-0.098})$=0.0155.

To compare the interval efficiency of DMU18 and DMU8, the interval efficiency of DMU18 is higher than that of DMU8 by 0.0432.